Mostly Sunny: A Forecast of Tomorrow's Power Index Research

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Abstract

Power index research has been a very active field in the last decades. Will this continue or are all the important questions solved? We argue that there are still many opportunities to conduct useful research with and on power indices. Positive and normative questions keep calling for theoretical and empirical attention. Technical and technological improvements are likely to boost applicability.

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1 Introduction

The 750-page tome "Power, Voting, and Voting Power: 30 Years After" which was edited by Holler and Nurmi (2013) demonstrates that the last three decades of research on power indices have been very productive. Can this go on? Or, as Manfred J. Holler put it: "Is there a future to power index research?" – addressing a scientific community that has seen several protagonists nominally retire of late.

The fact that two of us have only started to do research on power indices in the 2010s attests to our firm conviction that there is. There exists a set of diverse topics on which progress can still be made, and will be made.

The two recent articles on allocating voting weights in two-tier systems which have been published the most prominently (Barberá and Jackson, 2006; Koriyama et al., 2013) barely mention classical power measures. This may be regarded as a dark cloud in the sky of power index research. Top economics journals are

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concerned first and foremost with the welfare properties of voting systems. Power comes as a distant second or even third (behind epistemic concerns). But welfarist approaches to voting, which focus on measures of success rather than pivotality, can be viewed as part of power index research defined in a sufficiently liberal way. Moreover, we see no evidence that voting power faces greater suspicion from mainstream economists today than in the past.

We are convinced that today's prospects for power index research are no worse than 30 years ago. Our academic weather forecast is therefore: mostly sunny! Power index research will have a productive future. The specific topics which we expect to be addressed can be grouped loosely into three areas. In Section 2, we focus on the positive analysis of voting bodies. We then adopt a more normative, design-oriented perspective in Section 3. A range of technical issues for which progress is likely are discussed in Section 4. We conclude in Section 5.

2 Positive Analysis

Voting is important for the lives of billions of people. It shapes democratic participation at all levels of legislature and matters for decision making in boards or committees in the workplace. It also plays a role in non-governmental organizations, sports associations, and possibly even the decision on the next family trip (e.g., Darmann et al., 2012). As soon as voting and collective decision making come with a minimum of structure, power indices turn out to be useful.

More countries seem to adopt rather than abandon democratic governance structures, multinational organizations gain importance and decision-making bodies which use weighted voting evolve or are even newly created (see, e.g., Belke and Styczynska, 2006, on the Governing Council of the European Central Bank). Modern communication technology facilitates the coordination of geographically dispersed actors in associations and interest groups. Such organizations rely more and more on formal decision rules compared to consensus over coffee or beer. Reform suggestions for the most usual suspects for applications of power indices – the Council of the EU, the UN Security Council, the Board of Governors of the IMF – show no signs of fading. It is hence easy to affirm: the use of power indices in applied studies will continue. Some type of power index analysis is necessary in order to discover unevenness of the democratic playing field, which may be hidden behind vectors of weights, veto rules, thresholds, and quorums; it is also needed in order to assess rule changes.

We predict that old distinctions and divisions in the literature will lose importance, however. For instance, there exists a wide spectrum between (a) puristic *a priori* analysis, which purposely ignores any preference patterns of the past in favor of the far-reaching independence and symmetry assumptions that underlie the Penrose-Banzhaf index (PBI) or Shapley-Shubik index (SSI; see Felsenthal and Machover, 1998, or Laruelle and Valenciano, 2008a, for definitions and discussion), and (b) *a posteriori* analysis which places specific voters, say, individual members of the US Congress or Supreme Court, on locations in a multidimensional policy space in order to identify the critical Senators or judges for a given decision. Many normative studies of two-tier voting systems take corre-

lation between members of the same constituency behind the constitutional veil of ignorance. Why not do the same in positive analysis of, say, the IMF or EU?[1] The "veil of ignorance" is the most prominent motivation for independence and symmetry presumptions. But some asymmetries other than voting weights are often part of the game. For instance, some EU members use proportional and others first-past-the-post systems in order to determine their Council delegates. Some members of the IMF have preferential trade agreements or even share the same currency, others not. This deserves to be accounted for. To some extent, power indices based on games with a priori unions or a restricted communication structure have always held a middle ground between pure *a priori* and *a posteriori* analysis (see Owen, 1977, and Myerson, 1977, for pioneering work). But we see scope for more. And we predict that increased public transparency and improved technology for analyzing voting data will create a bias towards the *a posteriori* end of the range.[2]

Other dichotomies will also very fruitfully be replaced by a more pluralistic approach. Helpful as binary distinctions like *a priori* and *a posteriori*, full approval vs. rejection, P-power vs. I-power, take-it-or-leave-it committees vs. bargaining committees, etc. may be, they always narrow one's perspective. The attempt, e.g., to delineate the power to influence a collective decision ("I-power" in the for some time widely followed terminology of Felsenthal and Machover, 1998) from the power to appropriate the surplus or "prize" generated by it ("P-power") is certainly praiseworthy. But the seemingly crisp juxtaposition blurs the fact that both are intertwined, i.e., the distinction is fuzzy at best. It can therefore be highly misleading to base a categorization of available power indices on it.[3] It also makes a difference whether a decision making body can only adopt or reject an exogenous proposal (classified as a "take-it-or-leave-it committee" by Laruelle and Valenciano, 2008a) or if committee members bargain in search of agreement over a set of feasible alternatives (a "bargaining committee" according to Laruelle and Valenciano). But it makes a similarly big difference whether the proposals that are fed into a take-it-or-leave-it committee are composed strategically by an agenda setter who knows committee members' interests or whether they are truly exogenous; or whether the set of feasible alternatives that are negotiated in a bargaining committee is binary (declare independence or not), one-dimensional (tax rates, emission thresholds) or higher-dimensional (division of a monetary surplus).

With less "dichotomism" and a yet more diverse set of tools, future power index research will be better prepared to analyze the diverse voting bodies in the field. Ternary voting games (Felsenthal and Machover, 1997) allow more accurate positive analysis of, say, power in the UN Security Council; quaternary dichotomous voting rules (Laruelle and Valenciano, 2012) provide yet more flexibility.

[1] Kaniovski (2008) has made promising progress in this direction.

[2] See, for instance, the use by Badinger et al. (2013) of web scraping tools that are provided at http://api.epdb.eu/ in order to gather a data set of almost 70,000 individual voting decisions of EU member states on more than 3,000 proposals.

[3] For instance, the PBI is commonly classified as a measure of I-power but also captures P-power in some situations (see Felsenthal and Machover, 1998, p. 45). The SSI is frequently classified as a measure of P-power but also captures I-power in relevant contexts (see Napel and Widgrén, 2008; Kurz et al. 2014a).

Still more general frameworks for measuring power as pivotality or as outcome sensitivity have been suggested by Bolger (1993) and Napel and Widgrén (2004).

The latter framework is suited also to analyzing collective decision-making in sequential legislative procedures, which involve strategic interaction between the relevant players. The so-called "ordinary legislative procedure" of the European Union, formerly referred to as "codecision procedure", has proposals made or amended by three different voting bodies in several readings and the possibility of bargaining in a "conciliation committee". Positive analysis of the balance of power between European Commission, individual members of the Council, and the European Parliament therefore requires more than, say, a PBI calculation.[4]

The fact that conventional indices like the PBI or SSI are so much more convenient to compute has probably biased applied research in their favor – to the detriment of more complicated but perhaps more appropriate methodology. This adverse fate has presumably also affected the nucleolus of voting games. Montero (2006) has provided a very convincing motivation for its use as a power measure when bargaining takes place in the shadow of a voting rule. To our knowledge, however, its application to the EU Council by Le Breton et al. (2012) has been the first and only. Fortunately, given that we expect progress on the computational ease of power index research (see Section 4), we predict a brighter future for both the nucleolus and strategic analyses of voting procedures.

The blunt question "Which is the right power index?" has fortunately been replaced by more subtle ones, asking which of various properties that go with distinct indices or methods fit a specific application best. Different members of the community naturally differ in their answers. The Holler-Packel index (see Holler and Packel, 1983), for instance, is vigorously advocated by some while others group it under "minor indices" and hold that "any reasonable measure of a priori voting power ... must respect dominance" (which the Holler-Packel index does not – see Felsenthal and Machover, 2005; 1998, p. 245). Many scholars have expressed a pronounced preference for the PBI over the SSI at workshops and conferences; others have done the opposite.

This subjectivity and apparent arbitrariness is a cloud in the sky of power index research, at least from many outsiders' perspective. Fortunately, the literature has started to address the details of what constitutes power in which types of voting situations and what is the predictive value of power indices on a wider empirical basis. So far, laboratory experiments have been the method of choice. They provide maximal control over the aspects of a voting situation that determine a power index's potential value added. Montero et al. (2008), for instance, have conducted an experiment that empirically demonstrates the paradox of new members, which was a key prediction of power index analysis. Tentative support for the SSI and PBI has been found by Geller et al. (2004). More experimental power index research can be expected – someday perhaps even in the field.

A related area in which future empirical research could be promising is concerned with people's preferences for different voting systems. Can preferences for these be explained by the respective distribution of voting power, as measured by a particular index? How do people trade off procedural concerns (e.g.,

[4]See Mayer et al. (2013) on analysis of the codecision procedure for EU28, and Felsenthal et al. (2003, p. 490) on the "informational poverty" of traditional power indices.

for equal swing probabilities) and personal success propensities? Weber (2014) provides first evidence that subjects have a preference for voting systems that allocate Shapley-Shubik power to group representatives proportionally to group size. These systems are preferred over ones more in line with Penrose's square root rule to an extent that is not explicable by classic consequentialism.

3 Normative Analysis

The increased pluralism predicted for positive analysis has its natural analogues – and has in some cases been preceded by developments – in normative analysis. We already pointed to an improved account of given asymmetries in constitutional analysis. If, for instance, it is a restriction for the design of a two-tier voting system that the considered population partition must not be changed into constituencies of equal size, then it is appropriate to also take the reason for this restriction behind the veil of ignorance. More generally, power index research will do well to go beyond maximal symmetry and independence of voters.

Investigations of the "optimal" design of two-tier voting systems have branched into numerous different objective functions since the seminal investigation by Penrose (1946). Equality of voting power or of expected utility across individuals, maximal welfare under different utilitarian assumptions, minimal discrepancy between the outcomes of a two-tier vs. a direct voting system (with "discrepancy" operationalized by the probability of obtaining different outcomes or some notion of average outcome distance), and minimal discrepancy between weights and induced voting powers have all been considered.[5] The great majority of the studies have, however, remained faithful to Penrose's original binary setup, i.e., have considered a collective decision between two exogenously given alternatives (say, a random legislative proposal vs. the status quo). Neither voter abstention is considered nor the possibility of three or more ordered policy alternatives. Also the cases that binary proposals arise endogenously from strategic agenda setting or from two-party competition remain to be explored.

We forecast more departures from the conventional binary focus. There are still few: Laruelle and Valenciano (2008b) and Le Breton et al. (2012) have analyzed delegated bargaining over a simplex of policy alternatives, i.e., problems of rent division. Maaser and Napel (2007; 2012; 2014) have used Monte Carlo simulation in order to study influence-based, majoritarian, and welfarist objective functions in a median voter environment with an interval of policy options. Asymptotically optimal assignments of weights in the latter environment have been analytically characterized by Kurz et al. (2014a) for a democratic fairness objective similar to Penrose's. Because more than two policy alternatives give rise to population size effects on the distribution of delegate attitudes, it is surprising that the pattern obtained from binary setups has re-appeared also for a

[5] This list should still grow. Design of two-tier voting systems with epistemic goals or explicit minority protection constraints are promising research areas. It is also an open issue to cope with multiple normative criteria simultaneously. For instance, equitable representation in UNO or IMF can relate to countries' population sizes but also financial and other contributions to the common objective. No single "optimal rule" may exist; but which rules are Pareto-maximal with respect to a given set of criteria?

continuum of alternatives. Namely, optimal weights relate to the square root of population sizes in case of independent voters but plain proportionality is called for in case of at least mildly correlated constituency members. But the cases in between – with a finite number but more than two alternatives – have not been systematically studied so far. Preliminary computations indicate that the square root finding for independent and identically distributed (i.i.d.) voter attitudes may actually break down. Future research will clarify whether famous square root results are knife-edge not only with respect to their i.i.d. assumption but perhaps also with regard to allowing only two policy options.

A one-dimensional interval of alternatives already allows to analyze economic questions that would otherwise not be covered (e.g., scope of regulation, spending on climate change mitigation, monetary policy); it would be desirable to extend the analysis to multidimensional spaces. Future research in this vein will have to deal with the "curse of multidimensionality". One possibility could be to use point solutions, like the Copeland winner, which exist even if the generalized median voter does not. Another possibility is to assume an exogenous ordering of dimensions on which individuals vote sequentially (see De Donder et al., 2012).

So far, power index research and its normative applications to representative democracy have stayed closely in the tracks of winner-takes-all systems, which are easily modeled by weighted voting games. Other democratic systems like proportional rule or mixed-member systems have been neglected. We forecast that this will change. Edelman (2004), for instance, has considered the ideal composition of a legislature that contains representatives from equipopulous districts and some number of at-large representatives if the objective is to maximize the total Banzhaf power of individual citizens. Other scenarios with two (or even more) types of legislators, representing different interests of the electorate, are conceivable and will be studied in the future. What, for instance, should a mixed-member legislature or a two-chamber legislature ideally look like if voters have interests along ethnic and economic dimensions, which can be either independent or aligned in complicated ways?

4 Tools and Technical Issues

As in research more generally, the types of power investigations carried out depend on the available mathematical and computational tools. Substantial progress has been made regarding the efficient computation of power indices. Free software packages make it easy to calculate power indices for applied researchers who do not want to write their own programs; it is possible to adapt published code to a specific application (see, e.g., Macé and Treibich , 2012).

Understandably, the availability of software is biased towards the most popular conventional indices, namely the PBI and the SSI. But popularity is also a consequence of availability. We are unaware, for example, of any online tool which would allow an applied researcher to compute the nucleolus. For a 27-member assembly, as considered by Le Breton et al. (2012), its computation is an almost insurmountable obstacle for non-experts. So we see a future for more easy-to-use software, especially for the computation of technically more demand-

ing constructs (as, e.g., also the minimum sum representation index recently introduced by Freixas and Kaniovski, 2014). For power analysis based on convex policy spaces, algorithmic considerations are still in their infancy.

There is room for improvements even in the computation of SSI and PBI. Namely, the efficiency of the most widely used generating function approach (see Alonso-Meijide et al., 2012) relies heavily on working with small integer weights. This is in stark contrast with population figures in the millions being used as weights in the EU's Council. Large weights can also arise when trying to implement Penrose's square root rule as well as possible. Techniques have recently been developed to compute equivalent representations with smaller or even the minimum integer weights (see, e.g., Kurz, 2012a). These may in the future prove worthwhile for index computations, too.

Another important technical issue is the so-called "inverse problem" of power indices: for a given target distribution of power according to, say, the PBI or the SSI, one seeks to find a voting rule which induces this distribution as closely as possible for a given notion of distance. If one does not want to rely on simple heuristics, which mostly lack provable qualities such as a known maximal distance to the optimal solution, the problem is computationally very expensive (see De et al., 2012, and Kurz, 2012b). Progress can still be made regarding a better understanding of common heuristics (Kurz and Napel, 2014) and regarding the efficient – ideally also user-friendly – implementation of exact algorithms. The usefulness of, e.g., the integer linear programming techniques employed by Kurz (2012b) will benefit from steadily improving computer hardware; it is also conceivable that the complete list of distinct weighted voting games with up to nine players will in coming years become searchable online.

We also forecast progress in the pure theory of power indices. The distribution of inducible power vectors within the unit simplex is, also for the classical PBI or SSI, more mystery than understood. In a seminal recent paper, Alon and Edelman (2010) have shown that even for large numbers of players some target PBI distributions can be reached only with a large and constant relative error. Their work is in the process of being extended to other power indices (see Kurz, 2014).

Another theoretical issue of practical relevance is the possible coincidence of voting weights and power – either in an exact or asymptotic sense. It was shown only recently that the nucleolus of non-oceanic weighted majority games converges to the relative weight distribution (see Kurz et al., 2014b). The same article provided a new sufficient condition for exact coincidence of nucleolus and weights, which future research can presumably weaken. Coincidence of power and weights has also been studied recently by Houy and Zwicker (2014) for the PBI. Analogous findings for the SSI remain to be developed. The first attempt by Leech (2013) to develop an asymptotic result for power indices which covers both oceanic and non-oceanic games has turned out to misstate rather than generalize findings by Lindner and Machover (2004). But the goal was worthwhile, and we forecast that it will be achieved in future research.

5 Concluding Remarks

Above selection of topics for which we expect power index research to remain fruitful is biased by our own curiosities. That the collection is obviously too big an agenda for us alone, however, indicates the wide scope for continuing with or moving into power index research.

This scope becomes even wider if one also considers topics that are more distantly related to voting power. For instance, the quantifications of causal responsibility by Braham and van Hees (2009) or Felsenthal and Machover (2009) draw more or less explicitly on power analysis of non-strategic binary voting. Carrying methods and insights from non-binary strategic voting over into this domain looks promising. The domain of conventional power index research has also been left by Koster et al.'s (2014) investigation of the predictive value of knowing an individual voter's decision or voting inclination. Taking the latter as input into a model of an opinion formation process could merge traditional power analysis with the analysis of social dynamics and networks.

Finally, indices and techniques that have been popularized by voting applications can prove useful in completely unrelated contexts. For example, Kovacic and Zoli (2013) compute the PBI with relative population shares of different ethnicities as "weights" in an analysis of ethnic conflict. They find that a PBI-based approach can explain onset of conflict better than using existing indices of ethnic diversity.

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